Scarlet's Letters

Scarlet yawned. That was the last question, and it had only been half an hour. Not much of an exam. Looked like she'd have nothing to do for the *next* half hour. Well, it was second-year astronomy. What did she expect? She hadn't even needed her calculator.

Oops. That wasn't right. There'd been a multiple-choice question she'd put off for later. Just as simple as the rest: plugging numbers into an equation. For the other questions, she'd taken advantage of the fact that each answer had radically different values. She could just multiply significant figures and add powers of ten, put them together, and get the rough number. She didn't need to do precise calculations when she only needed to pick from answers that differed by factors of ten! Scarlet often wondered why professors even bothered having students plug numbers in. Letters were easier to remember and think about – if you had to actually *build* something you could always plug everything into a calculator. That's all everybody else bothered doing already.

This last question was no different. It asked for the energy, in Joules, carried by a photon with a wavelength of ten Angstroms. The equation was trivial: energy was proportional to photon frequency by a proportion of "h", Planck's constant.

$$E = h\nu$$

Frequency and wavelength of any wave multiplied together give the propagation speed of the wave, which, being light, is always "c". Well, not always – Scarlet had wondered for a moment if she should be worried the problem didn't explicitly say the photon was propagating in vacuum. Nonsense: they had not once discussed light propagating through any more optically dense medium in this class; Dr. Reinhart wasn't likely to introduce it on the exam. So:

$$\nu\lambda = c$$
$$\therefore \nu = \frac{c}{\lambda}$$
$$\therefore E = \frac{hc}{\lambda}$$

Scarlet reflected that many of her classmates probably memorized that last relation by rote and hadn't even had to spend the two seconds re-deriving it every time it came up, but Scarlet didn't mind the extra challenge; she could afford to be lazy.

The only problem was, she'd always been lazy about remembering the fundamental constants, too. The wavelength, λ, was given as 10 Å, or $10\cdot10^{-10}$ m in SI units. Even c was easy enough to remember: $3\cdot10^{8}$ m/s was close enough for her purposes. But she had no idea what the physical number for h was, not even its order of magnitude. It was, however, programmed into the calculator sitting in the backpack at her feet, along with many other things that would have shrunk the time the exam took to all of maybe ten minutes. Again, she liked the challenge. But she'd put that question off 'til now; she'd need the calculator to remember Planck's constant for her.

Scarlet leaned down to her backpack on the floor, carefully keeping her head up and eyes forward so Dr. Reinhart couldn't think they were wandering, and came back up with the calculator. She gracefully flipped off the lid and powered it on.

…and powered it on.

…uh oh. It wasn't coming on. Were the batteries dead? And then she remembered – she'd loaned the batteries to her roommate Grace last night, when Grace's Wii remote had died in the *middle* of a Super Smash Bros. brawl. She'd forgotten to requisition them afterwards. *Blast my generosity,* Scarlet laughed at herself.

Well, now what? Actually, most teachers provided a sheet of fundamental constants with the exam. Dr. Reinhart hadn't, but that may have just been oversight, and of course, she, like every other professor ever, seemed to love Scarlet. She could probably just go up to the desk and ask for Planck's constant without issue. Scarlet was about to do just that when she suddenly had a better idea:

Planck's is a proportionality *constant; I just need to remember the energy of* any other *frequency or wavelength of light and I can calculate* h *from that! I'm* sure *I've got the numbers I need* somewhere *in my head.* It wasn't long before she narrowed her memory combing to hydrogen. Several spectral lines were important; she ought to remember something in the Balmer series…visible light…H_α…656 nm!

But very shortly after, she realized she had no idea what its energy was. The only energy she could think of for hydrogen was its ionization energy, $13.6\ eV$. The two were related, of course: H_α was the first line in the Balmer series, which was a result of atoms reverting to an energy state of "n"=2. The equation would be simple enough, just involving a couple n^2's in the denominator. She could probably pull the equation out of her hat, but to her surprise she realized it wasn't necessary. For some odd reason, she could remember that the *Lyman* series of hydrogen, which involved energy transitions between n=1 and everything else, generated a "Lyman forest" of spectral lines, closer and closer together as the wavelength got less and less – corresponding to the energy transition being to/from higher and higher values of n – and the wavelengths – Scarlet had no idea why this number had stuck in her head, but it had – converged on 92 nm, just below the visible spectrum of light. That number corresponded to the transition from n=∞ to n=1; in other words, the re-ionization energy, $-13.6\ eV$.

So,

$$E = \frac{hc}{\lambda}$$

$$\therefore h = \frac{E\lambda}{c} = \frac{(13.6\ eV)(92 \cdot 10^{-9}\ m)}{c}$$

That would give Scarlet the value of h she needed to answer the question.

But there was another problem. Her energy-frequency relation she'd so brilliantly dreamed up used electron Volts, not Joules. Meaning her value for Planck's constant would be in $eV \cdot s$, not $J \cdot s$, the SI number she needed…

…If she had her calculator, she could just enter in whatever units she had and the calculator would do all the conversions along with the rest of the calculations. But of course if she had her calculator, she'd have just entered in "_h" ten minutes ago. No, she needed a way to relate Joules to electron Volts.

Well, she happened to remember they were related by a factor of "q", the charge of an electron. She couldn't recall instantly whether she should multiply or divide, but it didn't matter – she had no idea what the numerical value for q in SI units was. She needed something else. *Think*, she told herself, *think of something – anything – else to do with electron Volts.*

And it occurred to her: not one month before, she'd been doing a lab on measuring nuclear radiation in another class. The lab only really involved manipulating equipment and reading graphs, but of course Scarlet hadn't cared about that, so she'd pored over the theory behind it all. The basic premise was that unstable nuclei could eject positrons. They couldn't be measured directly because they annihilated almost instantly with electrons, generating two photons, traveling in opposite directions or however to conserve momentum. The photons each had an energy of $511\ keV$ – Scarlet wasn't sure why that number had stuck, either, but it had. And why that energy? Simple: to balance the annihilation of two particles with mass. The relation, a simple proportion *everybody* knew: $E=mc^2$. E was $511\ keV$, and "m" here was m_e, the mass of an electron or positron.

But how would that help? It didn't relate electron Volts to Joules! *Not directly*, Scarlet realized, *but I can plug everything in and then solve for "eV" like it's a variable, and get it in terms of only SI units, and that's all I need!*

$$511\ keV = 511 \cdot 10^3 eV = m_e c^2$$

$$\therefore eV = \frac{m_e c^2}{511} \cdot 10^{-3}$$

That is, Scarlet frowned, *if I had any idea what m_e is in SI units.*

Scarlet sat there, exasperated. She thought about giving up. But now she was having too much fun. She'd keep following what she could remember and maybe eventually she'd get to an equation where she actually *knew* the constants involved. What did she know about the mass of an electron? Well, it's much smaller than the total mass of an atom, for starters. As a matter of fact, Scarlet realized with a grin, she remembered from middle school that it was one over eighteen-hundred-something times the mass of a proton. Maybe that would help:

$$m_e = \frac{1}{1800} m_p$$

Dr. Reinhart announced "Ten minutes left," and Scarlet looked up with a start. She'd been working on this one problem for twenty minutes and the end was still not in sight! She heard several of her classmates groan. *Well,* she chided herself, *at least I'm lucky enough to have finished the rest of the test. I hope they don't have* too *much left.* She might as well do her best to finish this last problem. And if it took her the whole class, it'd be much easier to pretend she'd had as much trouble with it as everyone else.

So, what did she know about the mass of a proton? Actually, she felt like she could remember the order of magnitude for that one – about 10^{-16}. But no, perhaps that was its charge. After another few seconds' reflection, she decided she couldn't remember after all. She'd need something else. Anything.

And then she had it, and she knew she was almost done. A mole was defined by the number of carbon atoms – no, of only the ^{12}C isotope – in twelve grams of sample. There were six protons and six neutrons – they had essentially equivalent mass – in a ^{12}C nucleus, so the mass of a single ^{12}C atom was basically $12 \cdot m_p$, if you discount those pesky nuclear forces that make fusion a worthwhile endeavor. Now, a mole of these, or Avogadro's number of these, was equal to $12\ g$.

$$N_A \cdot 12 \cdot m_p = 12\ g$$

The twelves canceled. *Oh!* Scarlet realized to her shock, *Avogadro's number is just the number of protons in a gram, basically. That's so simple. How have I never realized that before?* But more importantly, it left her with

$$m_p = \frac{10^{-3} kg}{N_A}$$

the mass of a proton in SI units, if she knew Avogadro's number. And she did: $6.022 \cdot 10^{23}$.

Scarlet took a look at what she'd written so far:

$$E = \frac{hc}{\lambda}$$

$$h = \frac{(13.6\ eV)(92 \cdot 10^{-9}\ m)}{c}$$

$$eV = \frac{m_e c^2}{511} \cdot 10^{-3}$$

$$m_e = \frac{1}{1800} m_p$$

$$m_p = \frac{10^{-3} kg}{N_A}$$

Wow. I can derive all the important fundamental constants I don't already know just from a few random relations I happen to remember, and Avogadro's number of all things. The only one not here is q, but I can get that from "eV". Lemme think…

"Five minutes left." More groans.

Scarlet nearly laughed out loud. She'd almost forgotten she was taking an exam. She didn't have time to finish her new project; she'd have to figure it out after class. Right now she had to do the actual calculations. But that only took her a few minutes. She handed in her perfect exam, and as she walked out of class she allowed herself to indulge in the fantasy of meeting a *boy* who thought in any way similar to how she did.